A Research Proposal

On

A Study of Sufficient Criteria for Stability and Boundedness of Impulsive Differential Systems

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1. Introduction

Differential equations are essential tools in scientific modeling of physical problems which find relevance in almost every sphere of human endeavor from Agricultural sciences, Engineering, Medical science, and Physical sciences to Social sciences. In other way, many dynamical systems (Physical, Social, Biological, Engineering etc.) can be conveniently expressed in the form of differential equations[12]. In case of physical systems such as air craft's, some external forces acts which are not continuous with respect to time and the duration of their effect is near negligible as compared with total duration of original process. Same phenomenon's are observed in case of biological systems (e.g. heart beat, blood-flow, and pulse frequency), social systems (e.g. price-index frequency, demand and supply of goods) and in many other dynamical systems also such effects are called impulsive effects. Thus the theory of impulsive differential equations has wider scope and greater practical importance than the theory of differential equations without impulses because it is more closer to the practical problems. The theory of impulsive differential equations is now being recognized' to be not only richer than the corresponding theory of differential equations without impulses, but also represents a more natural framework for mathematical modeling of many real world phenomena.

In the development of the subject of differential systems with or without impulse, one may distinguish two broad distinct streams namely:

- An endeavor to obtain a definite or one of the definite types, either in closed forms, which are rarely possible or else by some process of approximation. This we shall refer to as the **Quantitative Theory** of differential systems.
- An endeavor to abandon all attempts to reach an exact or approximate solution, one strives to obtain information about the whole class of solution. This we shall call the **Qualitative Theory** of differential systems.

In our research proposal, it is proposed to investigate the following qualitative properties of the solution of the differential systems.

- **Stability**
- **Boundedness**

Stability and boundedness have great importance in the study of Differential systems. The mathematical models or equations that describe physical phenomena are in most cases differential systems of the form $\dot{x} = f(t, x)$, with the initial data $x(t_0) = x_0$. Since the

initial data, which often results from all types of measurements, may have errors, it is important to know the extent to which small disturbances in the initial data affect the desired behavior of the solutions of given system. If by making a sufficient small change in the initial data, a substantial deviation is observed in the corresponding solutions, then the solution obtained from the given initial data is unacceptable because it does not describe the required phenomena even approximately. The problem of investigating the conditions that will not allow the solutions to remarkably deviate from the desired behavior is therefore vital. The area of mathematics that deals with such problems relating to the behavior of the solutions is usually referred to as stability theory.

Similarly if the solution of the differential systems of the form $\dot{x} = f(t, x)$, with the initial data $x(t_0) = x_0$ is not bounded, means one of the solution of given system does not have finite value or approaching to infinite or diverges to infinite. So solution obtained from given initial data cannot be not acceptable. The problems of investigation the conditions that will not allow the solution to have infinite value or approaching to infinite or diverge to infinite has importance. This is refers to boundedness theory of the solution of differential systems.

Earlier, in stability and boundedness theory [1] we analyzed the behavior of the solutions using technique of the variation of the constant formula and integral inequalities. As a result, this analysis was confined to a small neighborhood of the operating point i.e. to stability in the small or local stability. Further, the techniques used require, in the case of linear systems, some explicit knowledge of solutions and in the case of weakly nonlinear systems, a complete grasp of the solution of the corresponding linear systems. These curbs apparently limit the application of the techniques when investigating the stability and boundedness behavior of a physical system.

A.M. Liapunov in 1892[1], introduced a completely different technique, known as Liapunov's second method , to determine the stability as well as boundedness behaviour of solutions of linear and nonlinear systems. The major advantage of this method is that stability and boundedness in the large can be obtained without any prior knowledge of solutions. Earlier this method used only to establish simple theorems on stability and boundedness. But from last 40 years his basic idea extensively exploited and effectively applied to entirely new problems in physics and engineering. Today, this method is widely recognized as an excellent tool not only in the study of differential equations but also in the theory of control systems, dynamical systems, systems with time lag, power

system analysis, time-varying nonlinear feedback systems, and so on. Its chief characteristic is the construction of a scalar function, namely, the Liapunov function. Unfortunately, it is sometimes very difficult to find a proper Liapunov function for a given system. Because the method yields stability information directly i.e. without solving the differential systems, it is also known as Liapunov's direct method.

In our research It is proposed to investigate the sufficient criteria for stability and boundedness of differential systems such as functional differential equations, Integro-differential equations and delay differential equations with impulsive by using Liapunov's direct method.

In the next part of the introduction it is given an overview of the mathematical tools used in the discussion of stability and boundedness, detail is as follows.

1.1 The basic description of systems with impulsive effect for fixed and variable time.

1.2 Mathematical formation of different differential systems with and without impulsive effect such as functional differential equations, Integro-differential equations and delay differential equations.

1.3 Basics definitions and theorems on stability and boundedness.

1.1 Description of System with Impulses

Let us consider an evolution process described by

(i) a system of differential equations

$$\dot{x} = f(t,x) \qquad (1.1)$$

Where f: $R_+ \times \Omega \to R^n$, $\Omega \subset R^n$ is an open set, R^n , the n-dimensional euclidean space and R_+, the nonnegative real line;

(ii) the sets M(t), N(t) $\subset \Omega$ for each $t \in R_+$; and

(iii) the operator A(t):M(t) $\to$ N(t) for each $t \in R_+$.

Let x(t)=x(t, t_0, x_0) be any solution of (1.1) starting at (t_0, x_0).The evolution process behaves as follows: the point P_t = (t, x(t)) begins its motion from the initial point $P_{t_0} = (t_0, x_0)$ and moves along the curve {(t , x):t $\geq$ t_0 , x=x(t)} until the time $t_1 > t_0$ at which the point P_t meets the set M(t). At $t=t_1$, the operator A(t) transfers the point $P_{t_1} = (t_1, x(t_1))$ into $P_{t_1^+} = (t_1, t_1^+) \in N(t_1)$ where $x_1^+ = A(t_1)x(t_1)$. Then the point P_t continues to move further along the curve with $x(t) = x(t, t_1, x_1^+)$ as the solution of (1.1) starting at $P_{t_1} = (t_1, x_1^+)$ until it hits the set M(t) at the next moment $t_2 > t_1$. Then, once

again the point $P_{t_2} = (t_2, x(t_2))$ is transferred to the point $P_{t_2^+} = (t_2, t_2^+) \in N(t_2)$ where $x_2^+ = A(t_2)\, x(t_2)$. As before, the point P_t continues to move forward with $x(t) = x(t, t_2, x_2^+)$ as the solution of (1.1) starting at (t_2, x_2^+). Thus the evolution process continues forward as long as the solution of (1.1) exists.

We shall call the set of relations (i), (ii) and (iii) that characterize the above mentioned evolution process an impulsive differential system, the curve which is described by the point P_t the integral curve and the function that defines the integral curve a solution of the impulsive differential system.

A solution of an impulsive differential system may be (a) a continuous function, if the integral curve does not intersect the set M(t) or hits it at the fixed points of the operator A(t); (b) a piecewise continuous function having finite number of discontinuities of the first kind if the integral curve meets M(t) at a finite number of points which are not the fixed points of the operator A(t); (c) a piecewise continuous function having a countable number of discontinuities of the first kind if the integral curve encounters the set M(t) at a countable number of points that are not the fixed points of the operator A(t).

The moments t_k at which the point P_t hits the set M(t) are called moments of impulsive effect. We shall assume that the solutions x(t) of the impulsive differential system is left continuous at t_k , k=1,2,..., that is, $x(t_k^-) = \lim_{h \to 0^+} x(t_k - h) = x(t_k)$

The freedom we have in the choice of the set of relations (i), (ii) and (iii) that describe an impulsive differential system gives rise to several types of systems. Let us discuss the following typical impulsive differential systems that are of interest.

1.1.1 Systems with impulses at fixed times.

Let the set M(t) represent a sequence of planes $t = t_k$ where $\{t_k\}$ is a sequence of times such that $t_k \to \infty$ as $k \to \infty$. Let us define the operator A(t) for $t=t_k$ only so that the sequence of operators $\{A(k)\}$ is given by

$$A(k) : \Omega \to \Omega, \quad x \to A(t)\, x = x + I_k(x)$$

where $I_k : \Omega \to \Omega$. As a result, the set N(t) is also defined for $t = t_k$ and therefore N(k)=A(k)M(k). With this choice of M(k), N(k) and A(k), a mathematical model of a simple impulsive differential system in which impulses occur at fixed times may be described by

$$\begin{cases} x' = f(t,x), & t \neq t_k, k = 1,2,..., \\ \Delta x = I_k(x), & t = t_k \end{cases} \qquad (1.2)$$

where for $t = t_k$, $\Delta x(t_k) = x(t_k^+) - x(t_k)$ and $x(t_k^+) = \lim_{h \to 0^+} x(t_k + h)$ We see immediately that any solution x(t) of (1.2) satisfies

(i) $x'(t) = f(t,x(t))$, $t \in (t_k, t_{k+1}]$, and

(ii) $\Delta x(t_k) = I_k(x(t_k))$, $t = t_k$, $k = 1,2, \ldots$ ·

The behavior of solutions is influenced by the impulsive effect.

1.1.2 Systems with impulses at variable times

Let {Sk} be a sequence of surfaces given by $S_k : t = \tau_k(x)$, k = 1, 2, ..., such that $\tau_k(x) < \tau_{k+1}(x)$ and $\lim_{h \to \infty} \tau_k(x) = \infty$. Then we have the following impulsive differential systems

$$\begin{cases} x' = f(t,x), & t \neq \tau_k(x) \\ \Delta x = I_k(x), & t = \tau_k(x), \; k = 1,2,... \end{cases} \qquad (1.3)$$

Systems with variable moments of impulsive effect such as (1.3) offer more difficult problems compared to the systems with fixed moments of impulsive effect. For example, note that the moments of impulsive effect for the system (1.3) depend on the solutions i.e., $t_k = \tau_k(x(t_k))$, for each k. Thus, solutions starting at different points will have different points of discontinuity. Also, a solution may hit the same surface $t=T_k(x)$ several times and we shall call such a behaviour "pulse phenomenon". In addition, different solutions may coincide after some time and behave as a single solution thereafter. This phenomenon is called "confluence".

1.2 Differential systems

1.2.1 Functional differential systems:

A functional differential equation is a differential equation in which the derivative $x'(t)$ of an unknown function x has a value at t that is related to x as a function of some other function at t. A general first-order functional differential equation is therefore given by $x'(t) = f(t, x(t), x(u(t)))$

1.2.2 Integro Differential systems:

An integro-differential equation is an equation which involves both integrals and derivatives of a function.The general first-order, linear integro-differential equation is of the form

$$u'(x) + \int_{x_0}^{x} f(t, u(t)\,dt = g(x, u(x)), \qquad u(x_0) = u_0, \quad x_0 \geq 0$$

1.2.3 Delay differential Systems:

A delay differential equation (also called a differential delay equation or difference_differential equation, although the latter term has a different meaning in the modern literature) is a special type of functional differential equation. Delay differential equations are similar to ordinary differential equations, but their evolution involves past values of the state variable. The solution of delay differential equations therefore requires knowledge of not only the current state, but also of the state a certain time previously.

Let us for the moment specialize further to equations with a single delay,

i.e. $x'(t) = f(x(t), x(t-\tau))$

The initial function would be a function $\mathbf{x}(t)$ defined on the interval $[-\tau;0]$

1.2.4 Impulsive Functional differential equations:

As we are familiar with Impulsive equations and functional differential equations ,we combine these two to get functional differential equations with impulses. The impulsive functional differential equations are adequate mathematical models of various real processes and phenomenon that are characterized by rapid change of their state and dependence on the pre history at each moment. These equations are a natural generalization of functional differential equations without impulses and of impulsive ordinary differential equations without delay.

If $\sigma \in R, A \geq 0$ and $x \in C([\sigma - r, \sigma + A], R^n)$ then for any

$t \in [\sigma, \sigma + A]$, Let $x_t \in C$ be defined by $x_t(\theta) = x(t + \theta), -r \leq \theta \leq 0$.

If $D \subset R \times C$, $f: D - R^n$ is given function. Then the system of impulsive functional differential equations is given by

$$\dot{x} = f(t, x), \qquad t \neq \tau_k(x(t)), t > 0.$$

$\Delta x(t) = x(\tau_k + 0) - x(\tau_k - 0) = I_k x(\tau_k), \quad t = \tau_k(x(t)), t > 0$, k=1,2,3……..

Here I_k are $n \times n$ matrices,

$\tau_k : R^n \to (t_0, \infty), \qquad t_0 = \tau_0 < \tau_1 < \cdots \ldots \ldots < \tau_k < \tau_{k+1} < \cdots \ldots \ldots$

and $\lim_{k\to\infty} \tau_k = \infty$

1.2.5 Impulsive Integro-differential equation:

As we are familiar with Impulsive equations and Intego-differential equations , we combine these two to get Integro differential equations with impulses.

Let us consider the linear Impulsive integro-differential equations.

$$x'(t) = A(t)\, x(t) + \int_{t_0}^{t} k(t,s)\, x(s)\, ds + F(t),\ t^1 t_k$$

$$\Delta x(t_k) = B_k\ x(t)\ ,$$

$$x(t_0^{+}) = x_{0,}$$

Where $0 \le t_0 < t_1 < t_2 < \ldots$ and t_k as $k \to \infty$,

$A \in PC^{+}[R_{+}, R^{n^2}], K \in PC^{+}[R_{+}^{2}, R^{n^2}], F \in PC^{+}[R_{+}, R^{n}]$ *and* $B_k \ge 0$ is a n x n matrix for each k such that $(I+B_k)^{-1}$ exists, I being the identity matrix,

1.2.6 Impulsive delay-differential equations:

As we are familiar with Impulsive equations and delay differential equations, we combine these two to get Impulsive delay differential equations. Since the theory of such equations has not yet develop sufficiently, Let us take an example of simple linear impulsive delay differential equation.

$$x'(t) = -p(t)\, x(t-\tau), t \ne t_k,$$

$$\Delta x(t_k) = b_k\ x(t_k),$$

$$x(0^{+}) = x_0,$$

$$x(t) = \phi(t),\ -\tau \le t < 0,$$

Where $p(t) \ge 0$ is continuous on $[0, \infty)$, $\tau > 0$, b_k are constants, ϕ is a given initial function and $0 < t_1 < t_2 < \ldots < t_k < \ldots$, with $\lim_{k\to\infty} \tau_k = \infty$

1.3 Basic Definitions of stabilities and Boundedness

Consider a system of differential equations.

$$\dot{X}(t) = f(t, X) \qquad (1.4)$$

Where X is an n-vector and f(t,X) is an n-vector function which is defined on a region $\Omega \subset I \times R^n$ (where I is an interval , a subset of R) and continuous in (t_0, X_0) so that for each (t_0, X_0) there is a solution $X(t; t_0, X_0)$ satisfying $X(t_0; t_0, X_0) = X_0$ and $X(t; t_0, X_0) = X$.

Definition 1.3.1

A solution X(t) to the system (1.4) is said to be **STABLE** if every solution $\overline{X}(t)$ of the system close to X(t) at initial time $t = 0$ remains close for all future time. In mathematical terms, reminiscent of the definition for a limit: For each choice of $\varepsilon > 0$ there is a $\delta > 0$ such that $|\overline{X}(t) - X(t)| < \varepsilon$ whenever $|\overline{X}(0) - X(0)| < \delta$. If at least one solution $\overline{X}(t)$ does not remain close, then X(t) is said to be unstable. Definitions says that for a stable solution you select the maximum amount of error ε you can tolerate between $\overline{X}(t)$ and X(t).The value δ, which depends on your choice of ε, tells you how close to X(0) you have to start in order to stay within that error. I refer to the stability of the system of differential equations as the physical stability of the system, emphasizing that the system of equations is a model of the physical behavior of the objects of the simulation.

Definition 1.3.2

A solution X(t) to the system (1.4) is said to be **ASYMPTOTICALLY STABLE** if it is stable and if there exists as stable $\delta_0 > 0$ such that $|\overline{X}(0) - X(0)| < \delta_0$ implies $|\overline{X}(t) - X(t)| \to 0$ as $t \to \infty$.

Definition 1.3.3

A solution X(t) to the system (1) is said to be **UNIFORMLY STABLE** if , for each $\varepsilon > 0$, there exists a $\delta = \delta(\varepsilon) > 0$ such that, for any solution $\overline{X}(t)$ of the inequalities $t_1 \geq t_0$ and $|\overline{X}(t_1) - X(t_1)| < \delta$ imply $|\overline{X}(t) - X(t)| < \varepsilon$ for all $t \geq t_1$.

Definition 1.3.4

A solution X(t) to the system (1) is said to be **UNIFORMLY ASYMPTOTICALLY STABLE** if it is uniformly stable and there is a $\delta_0 > 0$ and , for each $\eta > 0$, there exists a $T=T(\eta)>0$ such that the inequalities $t_1 \geq t_0$ and $|\overline{X}(t_1) - X(t_1)| < \delta_0$ imply $|\overline{X}(t) - X(t)| < \eta$ for all $t \geq t_1+T$.

Definition 1.3.5

A solution $X(t; t_0, X_0)$ of equation(1.4) is **BOUNDED** if there exists a $\beta > 0$ such that $| X(t; t_0, X_0) | < \beta$ for all $t \geq t_0$, where β may depend on each solution.

Definition 1.3.6

The solution of equation (1.4) is **EQUI-BOUNDED (EB)** if, for any $\alpha > 0$ and $t_0 \in I$, there exists a $\beta(t_0, \alpha) > 0$ such that if $X_0 \in S_\alpha$, where $S_\alpha = \{ x \in R^n : \|x\| < \alpha\}$, then $| X(t; t_0, X_0) | < \beta(t_0, \alpha)$ for all $t \geq t_0$, where α is a length of interval.

Definition 1.3.7

The solution of equation (1.4) is **UNIFORMLY BOUNDED** if, for any $\alpha > 0$ and $t_0 \in I$, there exists a $\beta(\alpha) > 0$ such that if $X_0 \in S_\alpha$, where $S_\alpha = \{ x \in R^n : \|x\| < \alpha\}$, then $| X(t; t_0, X_0) | < \beta(\alpha)$ for all $t \geq t_0$, where α is a length of interval

Definition 1.3.8

The solution of equation (1.4) is **ULTIMATELY- BOUNDED (UB)** for bound M, if there exist a $M > 0$ and for every solution $X(t, t_0, X_0)$ of (1.4) , there exist a $T = T(\alpha, X)$, such that $| X(t; t_0, X_0) | < M$ for all $t \geq t_0 + T$.

Definition 1.3.9

The solution of equation (1.4) is **UNIFORMLY ULTIMATELY BOUNDED (UUB)** for bound M, if there exist a $M > 0$ and if for any $\alpha > 0$ and $t_0 \in I$, there exist a $T(\alpha) > 0$, such that $X_0 \in S_\alpha$ implies that $| X(t; t_0, X_0) | < M$ for all $t \geq t_0 + T(\alpha)$.

Definition 1.3.10

If M in Definition 1.3.8 depends on t_0 and α i.e. $M(t_0, \alpha)$ for all t, then the solution of equation(1.4) is **EQUI-ULIMATELY BOUNDED**.

2. Review of Literature

The study of behavior of solutions of Differential Equations started in the latter part of the nineteenth century and became a subject of intense research since 1940.

Impulsive differential equations, by means, differential equations involving impulse effects, are seen as a natural description of observed evolution phenomenon of several real world problems. For examples, mechanical system with impact, biological phenomenon involving thresholds, bursting rhythm models in medicine and biology, optimal control models in economics, pharmacokinetics and industrial robotics and a host of others, do exhibit impulsive effect. So, it is beneficial to study the theory of impulsive differential equations in various fields in the future.

In addition to that problem of stability and boundedness of solutions holds a very significant place in the theory of impulsive differential equations

The study of certain ordinary differential equations with impulses was initiated in the 1960's by **Milman and Myshkis**[23]. They investigate the stability of the zero solution of differential equations with fixed moments of impulse actions by using the second Lyapunov method.

In 1989, **V. Lakshmikantham**[13], investigate the stability theory relative to a given solution of impulsive differential systems that played an important role in the development of complicated mechanism.

In 1991, **M. ramamohana Rao**[26], investigate sufficient condition for uniform stability and uniform asymptotic stability of impulsive integro differential equations by constructing suitable piecewise continuous Lyapunov-like functionals without the decresent property. A result which establishes no pulse phenomena in the given system is also discussed.

In 1994, **S. Kaul, V. Lakshmikantham and S. Leela**[10], investigates the existence of extremal solutions for IDE with variable times using a new approach, develop the necessary comparison result parallel to the one in ODE and apply it for the investigation of stability criteria. In the context of stability investigation, it is natural to consider the existence of a solution that meets each given barrier (hyper surface) exactly once, i.e. the lack of pulse phenomenon. With this motivation, they also investigate a result on the existence of solutions which meet the given hyper surfaces only once, and this result is a refinement of the some known results. The new idea of this paper will be of

value in the study of qualitative behavior of solutions of IDE with variable times whose progress so far was slow.

In 1998, **Jianhun shen and Jurang Yan**[28], investigate the sufficient conditions for stability of a general impulsive functional differential equations with fixed impulsive effects with the application of the concept of Liapunov-Razumikhin functions

In 2001, **J.S. Yu**[41], gives the stability for nonlinear delay differential equations of unstable type under impulsive perturbations. He investigated the sufficient conditions for stability of zero solution of said system, and shows that stability is caused by impulses. **Zhiguo Luo and Jianhun shen**[22], examine the implications of Lyapunov–Razumikhin-type stability and boundedness theorems for functional differential equations with infnite delays properly extended to impulsive functional differential equations using Lyapunov functional. Here some Razumikhin-type theorems on stability and boundedness for a class of impulsive functional differential equations with infinite delays are established. These theorems were rather general and therefore have great power in applications. **Xinzhi Liu and G. Ballinger**[17], investigated the sufficient criteria on uniform asymptotic stability for impulsive delay differential equations using Lyapunov functions and Raxumikhin techniques. It is shown that impulses do contribute to yield stability properties even when the underlying system does not enjoy any stability behavior. **Nikolai A and Olga S. chernikova**[25], investigate the sufficient conditions for the stability of integral sets. They introduced the concept of stability of integral sets for impulsive differential systems of a general type(with nonfixed times of impulse effect).

In 2002, **M. U. Akhmet**[2], investigate the sufficient criteria for stability, asymptotical stability and instability for non-trivial solutions of the impulsive systems by Lyapunov's second method. The construction of reduced system in the neighborhood of a nontrivial solution is a central auxiliary of his investigation.

In 2003, **Chunhai Kou, Shunian Zhang and Yongrui Duan**[11], investigates stability and instability properties in terms of two measures for impulsive delay differential equations with fixed moments of impulsive effects by variational Lyapunov Method. **Zhiguo Luo and Jianuhua Shen**[20], study the stability of a class of impulsive functional differential equations with infinite delays. In this paper they investigate a uniform stability theorem and a uniform asymptotic stability theorem, which shows that certain impulsive perturbations may make unstable systems uniformly stable, even uniformly asymptotically stable. **M.U. Akhmet**[3], investigates the sufficient criteria for

stability , asymptotical stability and instability of nontrivial solution of the impulsive differential system by Lyapunov's second method. The construction of a reduced system in the neighbourhood of nontrivial solution is a central auxiliary result of this paper.

In 2004, **Yu Zhang and Jitoa Sun**[42], investigates the practical stability of impulsive functional differential equations in terms of two measurements. In this paper they investigates some sufficient conditions of uniform practical stability for functional differential equations with impulses by using piecewise continuous Lyapunov functions and Razumikhin techniques.

In 2005, **Jurang Yan**[39], investigates the stability for the scalar impulsive delay differential equation, where delay arguments may be bounded or unbounded. He investigates the some new stability theorems which improve and extend several known results in the literature. **M Benchohra, J. Henderson and S.K. Ntouyas**[5], investigate the existence of solutions to some classes of impulsive functional and neutral differential equations with variable times and infinite delay. **Jianhua Shen and Jianli Li**[29], investigates the sufficient criteria on asymptotic stability for system of volterra functional differential equations with nonlinear impulsive perturbations using Lyapunov like functions with Razumikhin teachnique or Lyapunov like functional. It shown that given impulses do contribute to yield stability properties even when the underlying differential equation system does not enjoy any (or same) stability behavior. **S. J. Wu and D. Han**[34], first present the model of functional differential systems with impulsive effect on random moments. Then investigates the sufficient conditions for p-moments exponential stability of these systems by means of Liapunov's direct method coupled with Razumikhin technique and comparison principle.

In 2006, **Zhichun Yang and Daoyi Xu**[40], consider a class of nonlinear impulsive delay differential equations. They investigates several sufficient conditions ensuring the existence, uniqueness and global exponential stability of a periodic solution for nonlinear impulsive delay differential equations by By establishing, an exponential estimate for delay differential inequality with impulsive initial condition and employing Banach fixed point theorem. Furthermore, the criteria can be applied to analyze dynamical behavior of impulsive delay Hopfield neural networks and the results show different behavior of impulsive system originating from one continuous system. **Daoyi Xu, Wei Zhu and Shujun Long**[37], investigates some new sufficient conditions for global exponential stability of impulsive integro-differential equations by establishing an integro-differential, inequality with impulsive initial conditions and using the properties

of M-cone and eigenspace of the spectral radius of nonnegative matrices. **Yu Zhang and Jitao Sun**[43], consider the stability of impulsive infinite delay differential equations. they investigates some results on stability that are more general than ones given before using new technique that has been given by Shunian Zhang. They have extend this technique to study impulsive systems. **Bashir Ahmad and S. Sivasundaram**[2], investigates the sufficient conditions for the stability of the null solutions of impulsive hybrid set integro-differential equations with delay. **Huijun Wu and Jitao Sun**[35], investigates some stability criteria of p-moment stability for stochastic differential equations with impulsive jump and Markovian switching are obtained by using liapunov function method. **M.U. Akmet and M. Turan**[4], investigate differential equations on certain time scales with transition conditions(DETC) on the basis of reduction to the impulsive differential equations(IDE). **Jianhua shen, Jianli Li and Qing Wang**[30], investigate the boundedness and periodicity of solutions for impulsive ordinary differential equations and functional differential equations. They also gives two new boundedness theorems and new existence result for T- periodic solutions , which shows that impulsive perturbations do contribute to yielding periodic solutions even when the underlying systems do not enjoy periodic solution

In 2007 **Yu Zhang and Jitao Sun**[44], investigates some sufficient criteria of stability , asymptotic stability and practical stability for impulsive functional differential equations in which the state variables on the impulses are related to the time delay are provided by using Lyapunov functions and Razumikhin techniques. **Daoyi Xu, Zhichun Yang and Zhichun Yang**[38], investigate some extended and improve result on sufficient conditions ensuring the global exponential stability of the zero solution of an impulsive neutral differential equations by establishing a singular impulsive delay differential inequality and transforming the n-dimensional impulsive neutral delay differential equations to a 2n-dimensional singular impulsive delay differential equations. **Lin Wang and Xilin Fu**[33], investigates the stability properties in terms of two measures for impulsive differential systems with variable impulses. A new comparison principle which allows trajectories strike the some hyper surface finite times, is established and then be applied to obtain a stability criterion. **Zhang Chen and Xilin Fu**[7], investigate the stability of the trivial solution for impulsive functional differential system using several Lyapunov functions including partial components coupled with the Razumikhin technique and also obtained some new Razumikhin-type theorems which avoid using the auxiliary function P under less restrictive restrictive conditions.

Abdelghani Ouahab[24], investigate local and global existence and uniqueness results for first order impulsive functional differential equations with multiple delay and infinite delay. **Gani Tr. Stamov**[32], investigate if sufficient criteria for the stability of the moving invariant manifolds of nonlinear impulsive integro-differential equation using method of piecewise continuous Lyapunov's functions and the comparison principle.

In 2008, **Kaien Liu and Guowei Yang**[18], investigates the stability criteria in terms of two measures for impulsive functional differential equations via cone-valued Lyapunov functions and Razumikhin technique and also investigates the stability can be deduced form the (Q_0, Q)-stability of comparison impulsive differential equations. **X.J. Ran, M.Z. Liu and Q.Y. Zhu**[27], investigates the asymptotical stability of the numerical methods with the constant step size for impulsive differential equations. **Kaien Liu and Guowei Yang**[19], investigates the strict stability of impulsive functional differential systems by using Lyapunov functions and Razumikhin techniques and also given some comparison theorems by virtue of differential inequalities. **L. Berezansky and E. Braverman**[6], investigates the influence of impulsive perturbation of a linear impulsive equation in Banach space on the existence of bounded solutions and the exponential stability of the equations.

In 2009, **Sanjay K. Srivastava and Amanpreet kaur**[31], investigates new approach to stability theory of impulsive differential equations. They proposed that instead of putting all components of the state variable x in one Liapunov function, several functions of partial components of x, which can be much easier constructed, are used so that the conditions ensuring that stability are simpler and less restrictive. **Xiaodi Li**[14], investigates a new criterion on the uniform asymptotic stability and global stability for impulsive infinite delay differential equations by using Razumikhin technique and Lyapunov functions. This result is less restrictive and conservative than that given in some earlier references. It also shows that impulse can make unstable system stable. **Zhiguo Luo and Jianhua Shen**[21], investigates the stability of a class of impulsive functional differential equations by obtaining some general stability theorems with Liapunov functional. These results can be applied to finite delay impulsive systems or infinite delay impulsive systems or impulsive systems involving both finite and infinite delays, in a unified way.

In 2010, **Nor Shamsidah Bt Amir Hamzah, Mustafa Bin Mamat and J. Kavikumar**[7], investigates the algorithm which follows the theory of impulsive differential equations to solve the impulsive differential equations by using the second-

order Taylor series method. As many impulsive differential equations cannot be solved analytically or their solving is complicated. **S. G. Hristova**[8], investigates integral stability in terms of two measures for non-linear impulsive functional differential equations by using Razumikhin method, piecewise continuous Lyapunov functions and comparison scalar impulsive ordinary differential equations. The definition of integral stability in terms of two measures was introduced in this paper. **Snezhana G. Hristova**[9], defined a special type of stability combining two different measures and a dot product. The definition was a generalization of several types of stability known in the literature. He studied the stability of nonlinear impulsive differential equations with ``supremum''. He used Razumikhin's method as well as a comparison method for scalar impulsive ordinary differential equation. **Quanjun Wn, Jin Zhou and Lan Xiang**[36] , investigates the global exponential stability of a class of general impulsive retarded functional differential equations. Several new criteria on global exponential stability are analytically established based on Lyapunov function methods combined with Razumikhin techniques. **Xiaodi Li**[15], investigates sufficient criteria for global exponential stabilization of impulsive functional differential equations with infinite delays or finite delays by using Lyapunov functions and improved Razumikhin technique. The results obtained in this paper are helpful investigates the stability of control systems and synchronization control of chaotic systems. **Dongfang Li and Chengjian Zhang**[16],investigates nonlinear stability of discontinuous Galerkin methods for delay differential equations. They have introduced some concepts such as global and analogously asymptotical stability. These nonlinear stability properties reveal to the reader the relation between the perturbations of the numerical solution and that of the initial value or the systems.

3. Objective/Scope of the study

In the literature review, there is extensively study on the qualitative properties of differential system, where a sufficient criterion for stability and boundedness has been derived without impulse using Lyapunov's direct method. But much has not been done for the differential systems with impulsive effect. During our research project it is proposed to investigate the sufficient criteria for stability and boundedness of differential systems such as functional differential equations, Integro-differential equations and delay differential equations with impulsive effects using Lyapunov's direct method.

The thesis will be divided into three phases

Phase I: This phase includes the study of recent books and literature survey to collect the detailed information about the topic.

Phase II: The proposed research work will be done.

Phase III: The proposed work will be presented in the form of thesis. The first chapter of thesis will be introductory in nature. The remaining chapters will contain the new results regarding stability and boundedness of impulsive differential systems.

4. Proposed Methodology

4.1 Lyapunov's direct Method

Some qualitative properties stability and boundedness are investigated by several authors. For the study on stability and boundedness, most of these works were done by employing the solutions representations. Lyapunov (1892) dealt with stability and boundedness by two distinct methods, Lyapunov first and second method. The first method pre-supposes an explicit solution known and this is applicable to some restricted but important cases. As against this, the second method, which is also called the Direct method, is of great generality and power and above all does not require the knowledge of the solution themselves. The application of the Lyapunov method lies in constructing a scalar function (say V) and its derivatives (say $\dot{V}$) such that they posses certain properties. When these properties of V and $\dot{V}$ are shown, the stability and boundedness behavior of the system is known.

Following are some definitions for further references.

Definition 4.1.1: A function V(x) is called **POSITIVE DEFINITE** if V(0) = 0 and if around the origin V(x) > 0 for $x \neq 0$.

Definition 4.1.2: A function V(x) is called **NEGATIVE DEFINITE** if V(0) = 0 and if around the origin V(x) < 0 for $x \neq 0$.

Definition 4.1.3: A function V(x) is called **POSITIVE SEMI DEFINITE** if V(0) = 0 and if around the origin $V(x) \geq 0$ for $x \neq 0$.

Definition 4.1.4: A function V(x) is called **NEGATIVE SEMI DEFINITE** if V(0) = 0 and if around the origin $V(x) \leq 0$ for $x \neq 0$.

The direct method is via a special function called the Lyapunov function which define as follow.

Definition 4.1.5 A Lyapunov function V defined as $V : I \times R^n \rightarrow R$, is a real function of real variables X ($X \in R^n$) , t with the condition that $t \geq T$ and $|x_i| < H$. T and H are real constants of which T can be supposed to be as large as we wish and H as small as we wish but not zero having the following properties.

(i) Continuity: V(t,X) is continuous and single valued under the condition $t \geq T$ and $|x_i| < H$ and $V(t,0) = 0$.

(ii) V(t, X) is positive definite .

(iii) $\dot{V} = \frac{\partial V}{\partial x_1}\frac{dx_1}{dt} + \frac{\partial V}{\partial x_2}\frac{dx_2}{dt} \ldots + \frac{\partial V}{\partial x_n}\frac{dx_n}{dt}$ representing the total derivative with respect to t is negative definite.

Definition 4.1.6 (A Compete Lyapunov function) : A Lyapunov function V defined as $V: I \times R^n \to R$ is said to be **COMPLETE** if for $X \in R^n$,

(i) $V(t,X) \geq 0$

(ii) V(t, X) = 0 , if and only if X = 0 and

(iii) $\dot{V}(t,X) \leq -c|X|$ where c is any positive constant and $|X|$ given by $|X| = \left(\sum_{i=1}^{n}(x_i^2)\right)^{1/2} \to \infty$

It is **INCOMPLETE** if (iii) is not satisfied.

When the above properties of V and $\dot{V}(t,X)$ are shown, the qualitative behavior of the system can be discussed. The difficulty, however, arises when the necessary conditions cannot be exhibited; for then no conclusion can be drawn especially about the stability. Each problem is a new challenge, for the functions must be shaped a new for each given systems or class of systems. The proper choice of V depends to an extent upon the experience, ingenuity, and often, good fortune of the analyst.

Next some standard theorems on stability and boundedness of Lyapunov second method are given by using those we can investigate the sufficient criteria.

Let $\dot{X} = f(t, X)$ (4.1)

Where f: $I \times R^n \to R^n$ is a continuous n-vector function. Suppose that f(t,0) = 0 for all t, then the following theorem is true.

Theorem 4.1.7 : If the differential equations of undisturbed motion (the steady state of a system before perturbations are introduced) are such that it is possible to find a definite function V, of which the derivative $\dot{V}$ is a function of fixed sign, which is opposite to that of V or reduces identically to zero, the undisturbed motion is **STABLE**.

The following theorems are the various simplification of the Theorem 4.1.7

Theorem 4.1.8 : Assume that there exist a function V(t, X) defined for $t \geq 0$, $|X| < \delta_0$ (δ_0 is a positive constant) continuous with the following properties:

(i) $V(t, 0) = 0$

(ii) $V(t, X) \geq a(|X|)$, where a(r) is continuous monotonically increasing and a(0)=0.

(iii) $\dot{V}(t,X) \leq 0$, then the solution X(t) = 0 (zero solution) of equation (4.1) is **STABLE**

Theorem 4.1.9: Suppose conditions (i) and (ii) of Theorem 4.1.8 hold, and if we replace condition (ii) with (iv) $a(|X|) \leq V(t,X) \leq b(|X|)$, a(r) and b(r) being continuous monotonic increasing function and a(0) = b(0) = 0, then the zero solution of equation (4.1) is **UNIFORMLY STABLE(US).**

Theorem 4.1.10: Under the assumptions of the Theorem 4.1.8, if

(v) $\dot{V}(t,X) \leq -c|X|$, where c(r) is continuous on $[0, \delta_0]$ and positive definite, and if f(t,X) is bounded, then the zero solution of equation (4.1) is **ASYMPTOTICALLY STABLE(AS).**

Theorem 4.1.11: Under the same assumption of Theorem 4.1.9 with condition (v) of Theorem 4.1.10 , then the zero solution of Equation (4.1) is **UNIFORMLY ASYMPTOTICALLY STABLE(UAS).**

Theorem 4.1.12: If $\dot{V}(t,X) \leq -cV(t,X)$, where $c > 0$ is a constant under the same assumptions as in Theorem 4.1.9, then the zero solution of equation (4.1) is also **UNIFORMLY ASYMPTOTICALLY STABLE(UAS).**

Theorem 4.1.7 – 4.1.12 are Theorems on stability of the solution in the sense of Lyapunov with the use of Lyapunov function.

Following theorems are on the boundedness of the solution in sense of Lyapunov with the Lyapunov function.

Theorem 4.1.13: Suppose there exist a Lyapunov function V(t,X) defined on $I \times R^n$ which satisfies the following conditions:

(iv) $a(|X|) \leq V(t,X)$, where a(r) is continuous monotonically increasing and a(0)=0.

(v) $\dot{V}(t,X) \leq 0$, then the solutions of the equations(4.1) are **BOUNDED.**

Theorem 4.1.14: Suppose that there exists a Lyapunov function V(t, X) defined on $0 \leq t \leq R$, $|X| \geq R$ (where R may be large) which satisfies.

(i) $a(|X|) \leq V(t,X) \leq b(|X|)$, a(r) and b(r) being continuous monotonic increasing function, and

(ii) $\dot{V}(t,X) \leq 0$, then the solutions of equation (4.1) are **UNIFORMLY BOUNDED(UB) .**

Theorem 4.1.15: Under the assumptions of the Theorem 4.1.14 if $\dot{V}(t,X) \leq -cV(t,X)$, where c(r) is positive and continuous, then the solutions of equation (4.1) are **UNIFORM ULTIMALTELY BOUNDED(UUB).**

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